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Sound Radiation from Two Opposed Semi-Infinite Overlapped Ducts with Partial Lining

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Abstract

The radiation of plane sound waves from two opposed semi-infinite cylindrical duct with overlapping edges is investigated in the case the walls of the ducts lying in overlap region lined with different acoustically absorbent materials. By using the series expansion in overlap region and using Fourier transform technique elsewhere we obtain two uncoupled Wiener-Hopf equations their solutions involve unknown expansion coefficients satisfying a system of linear algebraic equations. Solving the algebraic system numerically by truncating the infinite series, the effect of the absorbent lining on the radiated field are presented graphically.

Key words: Sound radiation, absorbent lining, overlapping edges

1. Introduction

Propagation of waves in cylindrical ducts has frequently been an encountered topic for researchers when studying several engineering applications. Such as propagation of electromagnetic waves in coaxial cables and sound propagation in exhaust silencers. Due to sudden area changes in cross sectional area of the waveguide in such applications internal reflections occur and the energy in the transmitted wave decreases. Having such a geometry, simple expansion chambers have been shown to reduce the noise in car exhaust systems and widely investigated in literature [1], [2], [3], [4]. But, if there is a hole "outwards energy radiation" on the waveguide it will be very difficult to analyse the transmission properties of such an exhaust system. This problem was in depth examined in [5] with hard walled cylindrical ducts. In this paper, the radiation of sound waves from two opposed semi-infinite cylindrical ducts whose walls in finite overlap region are treated by different absorbent linings is investigated. So the objective of this paper is to analyse the radiation from the gap between two ducts and to reveal the influence of the partial lining on the radiated field. The difference from [5] is the partial lining in overlap region. This change in one hand has an importance in the application, on the other hand makes the problem very difficult (even impossible) to apply the same method used in [5]. To overcome this difficulty a hybrid method of formulation that employed previously in [6] is adopted in this paper. Expanding the field in the overlap region into a series of eigenfunctions and using the Fourier transform technique elsewhere the problem is reduced to two uncoupled Wiener-Hopf equations. Their solutions involve infinitely many unknown expansion coefficients satisfying a system of linear algebraic equations. Solving the algebraic system numerically the effect of the lining on the radiated field are presented graphically.

The time dependence is assumed to be $\exp(-i\omega t)$ with ω being the angular frequency and suppressed throughout this paper.

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2. Materials and Method

Consider two opposite semi-infinite circular cylindrical waveguides of different radii with common longitudinal axis, say z, in a cylindrical polar coordinate system (ρ, ϕ, z). While inner duct occupies the region $\rho = a$; $z < l$ and the outer duct occupies the region $\rho = b > a$; $z > 0$, where l represents the overlap length. The parts of the surfaces of the inner and outer ducts $\rho = a+0$ and $\rho = b - 0$ lying in the overlap region $0 < z < l$ are assumed to be treated by acoustically absorbing linings respectively. They are characterized by constant but different surface impedances $Z_1 = \frac{1}{\eta_1}, Z_2 = \frac{1}{\eta_2}$ and the remaining parts of the ducts are perfectly rigid (see Fig. 1). The ducts are immersed in an inviscid and compressible stationary fluid of density ρ_0 and sound speed c. A plane sound wave is incident from the positive z-direction, through the inner duct of radius $\rho = a$. From the symmetry of the geometry of the problem and the incident field the scattering field everywhere will be independent of the ϕ coordinate. We shall therefore introduce a scalar potential $u(\rho,z)$ which defines the acoustic pressure and velocity by $p = i\omega\rho_0 u$ and $\mathbf{v} = \text{grad } u$, respectively.

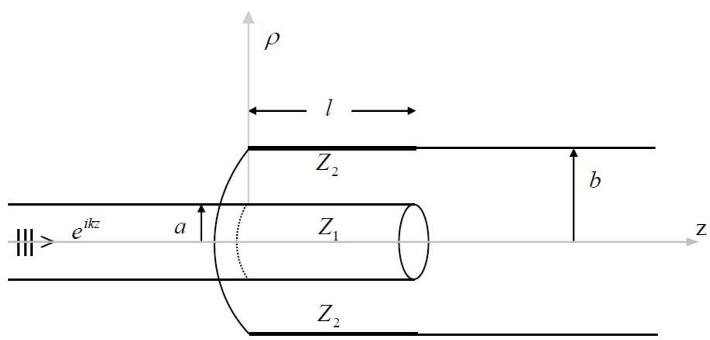

Figure 1. Geometry of the problem

Let the incident field be given by

$$u^i(\rho, z) = e^{ikz} \tag{1}$$

where $k = \omega/c$ denotes the wave number. For the sake of analytical convenience we will assume that the surrounding medium is slightly lossy and k has a small positive imaginary part. The lossless case can be obtained by letting $\text{Im}(k) \to 0$ at the end of the analysis.

It is convenient to write the total field in different regions as:

$$u^T(r,z) = \begin{cases} u_1(\rho,z) & , \quad \rho > b, z \in (-\infty,\infty) \\ u_2^{(1)}(\rho,z)H(-z) + & \\ \quad u_2^{(2)}(\rho,z)\,[H(z) - H(z-l)] + u_2^{(3)}(\rho,z)H(z-l), & a < \rho < b, z \in (-\infty,\infty) \\ u_3(\rho,z) + u^i(\rho,z) & , \quad \rho < a, z \in (-\infty,\infty) \end{cases} \tag{2}$$

where $H(z)$ is the unit step function.

2.1. Fourier transformation/Wiener-Hopf equations

The unknown scattered fields $u_1(\rho, z)$ and $u_2^{(1)}(\rho, z)$ satisfy the Helmholtz equation for the regions $\rho > b, -\infty < z < \infty$ and $a < \rho < b, -\infty < z < 0$, respectively

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{\partial^2}{\partial z^2} + k^2\right] \begin{matrix} u_1(\rho, z) \\ u_2^{(1)}(\rho, z) \end{matrix} = 0 \tag{3}$$

and are to be determined by the following boundary and continuity relations:

$$\frac{\partial}{\partial\rho} u_1(b, z) = 0 \quad , \qquad z > 0 \tag{4}$$

$$\frac{\partial}{\partial\rho} u_2^{(1)}(a, z) = 0 \quad , \qquad z < 0 \tag{5}$$

$$\frac{\partial}{\partial\rho} u_1(b, z) = \frac{\partial}{\partial\rho} u_2^{(1)}(b, z) \quad , \qquad z < 0 \tag{6}$$

$$u_1(b, z) = u_2^{(1)}(b, z) \qquad , \qquad z > l \tag{7}$$

Applying Fourier transform to the above mixed boundary value problem and making necessary arrangements, we first achieve an equation for $F(\rho, \alpha)$ being the Fourier transform of $u_1(\rho, z)$, as

$$F(\rho, \alpha) = -\dot{F}_-(b, \alpha)\frac{H_0^{(1)}(K\rho)}{K(\alpha)H_1^{(1)}(Kb)} \tag{8}$$

with $$F(\rho, \alpha) = \int_{-\infty}^{\infty} u_1(\rho, z)e^{i\alpha z}dz = e^{i\alpha l}[F_+(\rho, \alpha) + F_-(\rho, \alpha)] \tag{9}$$

and then we obtain the first decoupled Wiener-Hopf equation valid in the strip $Im(-k) < Im(\alpha) < Im(k)$, where α is the complex Fourier transform variable.

$$\frac{b}{2}F_+(b, \alpha) - \dot{F}_-(b, \alpha)\frac{L(\alpha)}{K^2(\alpha)} = -\frac{b}{\pi a}\frac{(i\alpha f_0 - g_0)}{K^2(\alpha)} + \frac{1}{\pi}\sum_{m=1}^{\infty}\frac{J_1(K_m a)}{J_1(K_m b)}\frac{(i\alpha f_m - g_m)}{\alpha_m^2 - \alpha^2} \tag{10}$$

here $L(\alpha)$ is kernel function defined and factorized by

$$L(\alpha) = \frac{H_1^{(1)}(Ka)}{\pi[J_1(Ka)Y_1(Kb) - J_1(Kb)Y_1(Ka)]H_1^{(1)}(Kb)} = L_+(\alpha)L_-(\alpha) \tag{11}$$

explicit form of split functions $L_\pm(\alpha)$ are given in [7] and $K(\alpha)$ is square root function defined as

$$K(\alpha) = \sqrt{k^2 - \alpha^2} \qquad , \tag{12}$$

The functions $F_\pm$ are half-plane analytical functions described by Fourier integrals as:

$$F_\pm(\rho, \alpha) = \pm\int_0^{\pm\infty} u_1(\rho, z)e^{i\alpha z}dz \tag{13}$$

where $\cdot$ denotes the derivative with respect to ρ.
Owing to the analytical properties of $F_{\pm}$, $L_{\pm}$ and following classical procedures of Wiener-Hopf technique we get the solution of equation in (10) of the form:

$$\dot{F}_{-}(b,\alpha)L_{-}(\alpha)=\frac{b}{\pi a}\frac{(ikf_0-g_0)}{L_{+}(k)}-\frac{(k-\alpha)}{\pi}\sum_{m=1}^{\infty}\frac{J_1(K_m a)}{J_1(K_m b)}\frac{(k+\alpha_m)}{L_{+}(\alpha_m)}\frac{(i\alpha_m f_m-g_m)}{2\alpha_m(\alpha_m-\alpha)} \tag{14}$$

where $K_m=K(\alpha_m),(m=1,2\ldots)$ are the zeros of the function $K^2(\alpha)[J_1(Ka)Y_1(Kb)-J_1(Kb)Y_1(Ka)]$ on upper half of complex α-plane. f_m, g_m are the expansion coefficients and satisfies a system of algebraic equations which will be given in the next sub-section.
Similarly, the unknown scattered fields $u_3(\rho,z)$ and $u_2^{(3)}(\rho,z)$ satisfy the Helmholtz equation for the regions $\rho<a,-\infty<z<\infty$ and $a<\rho<b, l<z<\infty$, respectively

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{\partial^2}{\partial z^2}+k^2\right]\begin{matrix}u_3(\rho,z)\\ u_2^{(3)}(\rho,z)\end{matrix}=0 \tag{15}$$

and are to be determined by the following boundary and continuity relations:

$$\frac{\partial}{\partial\rho}u_3(a,z)=0\ ,\qquad z<l \tag{16}$$

$$\frac{\partial}{\partial\rho}u_2^{(3)}(b,z)=0\ ,\qquad z>l \tag{17}$$

$$\frac{\partial}{\partial\rho}\left[u_3(a,z)+u^i(a,z)\right]=\frac{\partial}{\partial\rho}u_2^{(3)}(a,z)\ ,\ \ z>l \tag{18}$$

$$u_3(a,z)+u^i(a,z)=u_2^{(3)}(a,z)\ ,\quad z>l \tag{19}$$

Taking Fourier transform of the Helmholtz equation (15) together with the relations (16-19) and making necessary arrangements we arrive at the second decoupled W-H equation to be solved,

$$\dot{H}_{+}(a,\alpha)\frac{N(\alpha)}{K^2(\alpha)}+\frac{a}{2}H_{-}(a,\alpha)=-\frac{a}{\pi b}\frac{(p_0-i\alpha q_0)}{K^2(\alpha)}+\frac{1}{\pi}\sum_{m=1}^{\infty}\frac{J_1(K_m b)}{J_1(K_m a)}\frac{(p_m-i\alpha q_m)}{\alpha_m^2-\alpha^2}-\frac{a}{2}\frac{e^{ikl}}{i(k+\alpha)} \tag{20}$$

where $H_{\pm}(\rho,\alpha)$ are half-plane analytical functions on complex α-plane defined by Fourier integrals as:

$$H_{\pm}(\rho,\alpha)=\pm\int_{l}^{\pm\infty}u_3(r,z)e^{i\alpha(z-l)}dz \tag{21}$$

$N(\alpha)$ in (20) stands for the kernel function

$$N(\alpha)=\frac{J_1(Kb)}{\pi\left[J_1(Ka)Y_1(Kb)-J_1(Kb)Y_1(Ka)\right]J_1(Ka)} \tag{22}$$

and will be factorized as

$$N(\alpha)=N_{+}(\alpha)N_{-}(\alpha)\qquad , \tag{23}$$

and their explicit expressions can be found in [8].
Applying standard factorization and decomposition procedures to the equation (20) together with Liouville's theorem we get the W-H solution of the form:

$$\dot{H}_+(a,\alpha)\frac{N_+(\alpha)}{(k+\alpha)} = -\frac{a}{\pi b}\frac{(p_0+ikq_0)}{(k+\alpha)N_+(k)} + \frac{1}{\pi}\sum_{m=1}^{\infty}\frac{J_1(K_m b)}{J_1(K_m a)}\frac{(k+\alpha_m)}{N_+(\alpha_m)}\frac{(p_m+i\alpha_m q_m)}{2\alpha_m(\alpha_m+\alpha)} - \frac{ka\,e^{ikl}}{i(k+\alpha)N_+(k)} \quad (24)$$

where p_m, q_m's are expansion coefficients and will be determined later.

2.2. Series Expansion and Determination of Unknown Coefficients

The unknown field $u_2^{(2)}(\rho,z)$ satisfies Helmholtz equation in the region $a < \rho < b$, $0 < z < l$

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{\partial^2}{\partial z^2}+k^2\right]u_2^{(2)}(\rho,z)=0 \quad (25)$$

together with the relations:

$$\left[ik\eta_1+\frac{\partial}{\partial\rho}\right]u_2^{(2)}(a,z)=0 \ , \qquad 0<z<l \quad (26)$$

$$\left[ik\eta_2-\frac{\partial}{\partial\rho}\right]u_2^{(2)}(b,z)=0 \ , \qquad 0<z<l \quad (27)$$

$$\frac{\partial}{\partial z}u_2^{(1)}(\rho,0)-\frac{\partial}{\partial z}u_2^{(2)}(\rho,0)=0 \ , \qquad a<\rho<b \quad (28)$$

$$u_2^{(1)}(\rho,0)-u_2^{(2)}(\rho,0)=0 \ , \qquad a<\rho<b \quad (29)$$

$$\frac{\partial}{\partial z}u_2^{(3)}(\rho,l)-\frac{\partial}{\partial z}u_2^{(2)}(\rho,l)=0 \ , \qquad a<\rho<b \quad (30)$$

$$u_2^{(3)}(\rho,l)-u_2^{(2)}(\rho,l)=0 \ , \qquad a<\rho<b \quad (31)$$

So it can be expressed in terms of the waveguide modes as

$$u_2^{(2)}(\rho,z)=\sum_{n=0}^{\infty}\left[a_n e^{i\beta_n z}+b_n e^{-i\beta_n z}\right]\left[J_0(\gamma_n\rho)-R_nY_0(\gamma_n\rho)\right] \quad (32)$$

with

$$R_n=\frac{ik\eta_1J_0(\gamma_n a)-\gamma_nJ_1(\gamma_n a)}{ik\eta_1Y_0(\gamma_n a)-\gamma_nY_1(\gamma_n a)}=\frac{ik\eta_2J_0(\gamma_n b)+\gamma_nJ_1(\gamma_n b)}{ik\eta_2Y_0(\gamma_n b)+\gamma_nY_1(\gamma_n b)} \quad (33)$$

where γ_n's are the roots of the equation

$$\frac{ik\eta_1J_0(\gamma_n a)-\gamma_nJ_1(\gamma_n a)}{ik\eta_1Y_0(\gamma_n a)-\gamma_nY_1(\gamma_n a)}-\frac{ik\eta_2J_0(\gamma_n b)+\gamma_nJ_1(\gamma_n b)}{ik\eta_2Y_0(\gamma_n b)+\gamma_nY_1(\gamma_n b)}=0 \quad (34)$$

while β_n's are defined as

$$\beta_n=\sqrt{k^2-\gamma_n^2} \ , \quad n=1,2,\ldots \quad (35)$$

Taking into account continuity relations (28-31) together with the expression (32) and W-H solutions (14), (24); we obtain a set of linear algebraic equations in terms of the unknown coefficients a_n, b_n which are related with f_m, g_m, p_m, q_m.

$$\frac{a\pi^2}{2b}L_+(k)\sum_{n=0}^{\infty}[a_n(k+\beta_n)+b_n(k-\beta_n)]\Delta_{0n}=$$
$$+\frac{b}{aL_+(k)S_0}\sum_{n=0}^{\infty}[a_n(k-\beta_n)+b_n(k+\beta_n)]\Delta_{0n}$$
$$-k\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}[a_n(\alpha_m-\beta_n)+b_n(\alpha_m+\beta_n)]\frac{\Delta_{mn}}{\alpha_m L_+(\alpha_m)S_m}\frac{J_1(K_m a)}{J_1(K_m b)} \quad (36)$$

$$\frac{\pi^2}{2}L_+(\alpha_r)\frac{J_1(K_r b)}{J_1(K_r a)}\sum_{n=0}^{\infty}[a_n(\alpha_r+\beta_n)+b_n(\alpha_r-\beta_n)]\Delta_{rn}=$$
$$+\frac{b}{aL_+(k)S_0}\sum_{n=0}^{\infty}[a_n(k-\beta_n)+b_n(k+\beta_n)]\Delta_{0n} \qquad (r=1,2,\dots)$$
$$-(k+\alpha_r)\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}[a_n(\alpha_m-\beta_n)+b_n(\alpha_m+\beta_n)]\frac{\Delta_{mn}}{2\alpha_m L_+(\alpha_m)S_m}\frac{J_1(K_m a)}{J_1(K_m b)}\frac{(k+\alpha_m)}{(\alpha_m+\alpha_r)} \quad (37)$$

$$\frac{\pi^2}{2}N_+(k)\sum_{n=0}^{\infty}\left[a_n(k-\beta_n)e^{i\beta_n l}+b_n(k+\beta_n)e^{-i\beta_n l}\right]\Delta_{0n}=$$
$$+\frac{1}{N_+(k)S_0}\sum_{n=0}^{\infty}\left[a_n(k+\beta_n)e^{i\beta_n l}+b_n(k-\beta_n)e^{-i\beta_n l}\right]\Delta_{0n}$$
$$-k\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left[a_n(\alpha_m+\beta_n)e^{i\beta_n l}+b_n(\alpha_m-\beta_n)e^{-i\beta_n l}\right]\frac{\Delta_{mn}}{\alpha_m N_+(\alpha_m)S_m}-\frac{\pi k a e^{ikl}}{N_+(k)} \quad (38)$$

$$\frac{\pi^2}{2}N_+(\alpha_r)\sum_{n=0}^{\infty}\left[a_n(\alpha_r-\beta_n)e^{i\beta_n l}+b_n(\alpha_r+\beta_n)e^{-i\beta_n l}\right]\Delta_{rn}=$$
$$+\frac{1}{N_+(k)S_0}\sum_{n=0}^{\infty}\left[a_n(k+\beta_n)e^{i\beta_n l}+b_n(k-\beta_n)e^{-i\beta_n l}\right]\Delta_{0n} \qquad (r=1,2,\dots)$$
$$-\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left[a_n(\alpha_m+\beta_n)e^{i\beta_n l}+b_n(\alpha_m-\beta_n)e^{-i\beta_n l}\right]\frac{(k+\alpha_m)\Delta_{mn}}{2\alpha_m N_+(\alpha_m)S_m}\frac{(k+\alpha_r)}{(\alpha_m+\alpha_r)}-\frac{\pi k a e^{ikl}}{N_+(k)} \quad (39)$$

where

$$S_0=\frac{2}{\pi^2}\frac{a^2-b^2}{a^2} \quad , \quad S_m=\frac{2}{\pi^2}\frac{J_1^2(K_m a)-J_1^2(K_m b)}{J_1^2(K_m b)} \quad (40)$$

$$\Delta_{0n}=\frac{2}{\pi\gamma_n}\left\{[J_1(\gamma_n a)-R_n Y_1(\gamma_n a)]-\frac{b}{a}[J_1(\gamma_n b)-R_n Y_1(\gamma_n b)]\right\} \quad (41)$$

$$\Delta_{mn}=\frac{2}{\pi}\frac{\gamma_n}{\gamma_n^2-K_m^2}\left\{[J_1(\gamma_n a)-R_n Y_1(\gamma_n a)]-\frac{J_1(K_m a)}{J_1(K_m b)}[J_1(\gamma_n b)-R_n Y_1(\gamma_n b)]\right\} \quad (42)$$

For radiated field we need to determine the coefficients $\{f_m, g_m\}$. Their relation with $\{a_n, b_n\}$ is

$$i\alpha f_0 + g_0 = \frac{i}{S_0}\sum_{n=0}^{\infty}[a_n(\alpha+\beta_n) + b_n(\alpha-\beta_n)]\Delta_{0n} \tag{43}$$

$$i\alpha f_m + g_m = \frac{i}{S_m}\sum_{n=0}^{\infty}[a_n(\alpha+\beta_n) + b_n(\alpha-\beta_n)]\Delta_{mn} \tag{44}$$

2.3. Radiated Field

The radiated field $u_1(\rho, z)$ can be obtained by taking the inverse Fourier transform of $F(\rho, \alpha)$. From (8) and (9) we can write,

$$u_1(\rho, z) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\dot{F}_-(b,\alpha)\frac{H_0^{(1)}(K\rho)}{K(\alpha)H_1^{(1)}(Kb)}e^{-i\alpha z}d\alpha \tag{45}$$

Taking into account the asymptotic expansion of $H_0^{(1)}(K\rho)$ when $k\rho \to \infty$

$$H_0^{(1)}(K\rho) \approx \sqrt{\frac{2}{\pi K\rho}}e^{i(K\rho-\pi/4)} \tag{46}$$

and using saddle point technique together with (14) we evaluate the integral in (45) for the radiated field as,

$$u_1(\rho, z) \approx \mathcal{F}(\theta)\frac{e^{ikr}}{kr} \tag{47}$$

with

$$\mathcal{F}(\theta) = \frac{i}{\pi^2}\frac{1}{L_+(kcos\theta)sin\theta H_1^{(1)}(kbsin\theta)}$$
$$\times\left(\frac{b}{a}\frac{(ikf_0 - g_0)}{L_+(k)} - (k + kcos\theta)\sum_{m=1}^{\infty}\frac{J_1(K_m a)}{J_1(K_m b)}\frac{(k+\alpha_m)}{L_+(\alpha_m)}\frac{(i\alpha_m f_m - g_m)}{2\alpha_m(\alpha_m + kcos\theta)}\right) \tag{48}$$

where (r, θ) are spherical coordinates defined as,

$$z = rcos\theta \quad , \quad \rho = rsin\theta \tag{49}$$

3. Results and Discussion

In order to show the effect of the absorbent lining characterized by the surface admittances $\eta_{1,2}$ on the sound radiation, some numerical results showing the variation of the amplitude $\mathcal{F}(\theta)$ of the radiated field are presented. In numerical calculations the solution of the infinite system of algebraic equations is obtained by truncating the infinite series at some number *N*, this number is dependent on the dimensionless duct radii *ka* and *kb*. Since the propagating mode number increases for greater value of *kb*, we have to choose truncation number *N* according to this situation. It is observed that the number *N* always must be greater than the propagating mode number in outer duct. For simplicity in numerical calculation we also limit ourselves with only imaginary values of surface admittances such that $\eta_{1,2} = iX_{1,2}$, $X \in \mathbb{R}$.

In figures (Fig. 2, 3, 4) the far field amplitude is demonstrated in polar plot with changing angle θ from 0 to π to see the effect of lining on radiation phenomenon. In all far field graphs some amount of decrease in the radiated field in all directions is observed in comparison with the hard walled duct. In Fig. 4 the parameter values are chosen as in the Fig. 6 of [5] and excellent agreement is observed for the "rigid" plot. This agreement verifies the correctness of the present method.

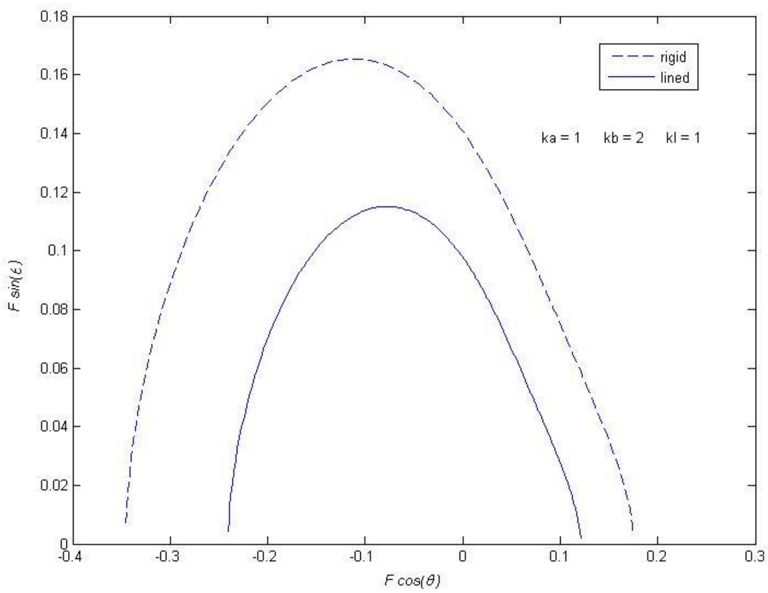

Figure 2. Polar plot of the far field amplitude function $\mathcal{F}(\theta)$.

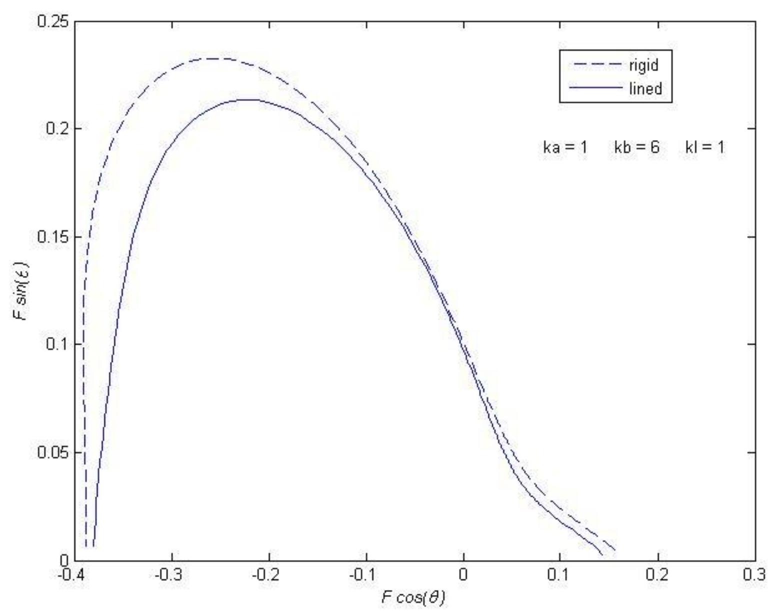

Figure 3. Polar plot of the far field amplitude function $\mathcal{F}(\theta)$.

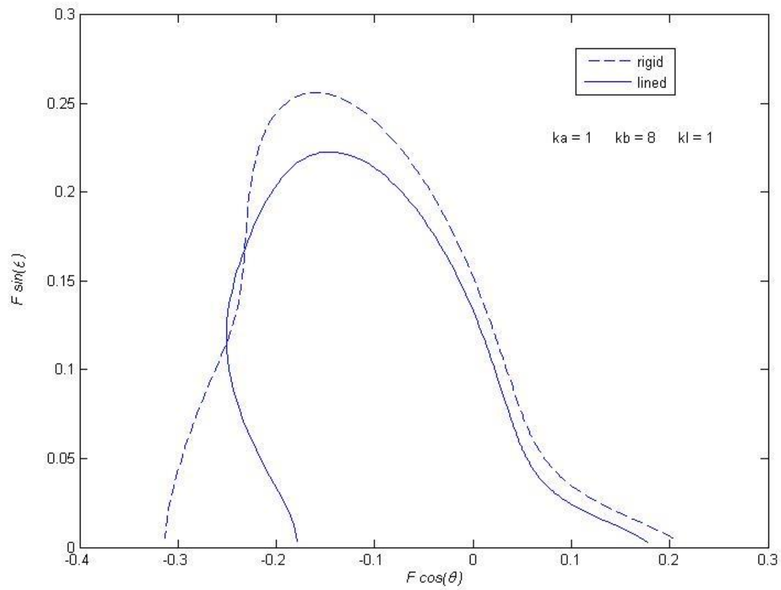

Figure 4. Polar plot of the far field amplitude function $\mathcal{F}(\theta)$.

Conclusions

The radiation of sound from two semi-infinite circular cylinders having a common axis but different radii is investigated. Two cylinders overlap in a finite region and in this region they have different linings on their walls. Using the mode matching method in overlap region and Fourier transform elswhere the well-known Wiener-Hopf technique successfully applied to the problem. The problem is first reduced to two decoupled Wiener Hopf equations and then solved following usual factorization and decompozition procedures. The solution involves two systems of linear algebraic equations involving two sets of infinitely many unknown expansion coefficients. Numerical solution to these systems is obtained for various values of the problem parameters such as overlap length *kl*, lining admittances $\eta_{1,2}$. In the case where the lining admittances are zero, the results obtained in this paper are compared with the results of [5] and the agreement is perfect. Furthermore, it is observed that choosing impedance values appropriately it is possible to attenuate radiation and so to increase transmission at the same time.

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